CHAPTER 11

Gaussian Beam Imagery

Coherent light generated by lasers has properties different from light generated by other sources which we usually deal with in more conventional optical systems. If we look through a telescope at a distant object, the light intensity across the entrance pupil and aperture stop is uniform, and this is generally known as a "top-hat" intensity profile or distribution. A telescope objective, if it is free of aberrations, focuses a point object into an Airy disk pattern, with the diameter determined by the f/number or numerical aperture of the objective lens. In this case, a uniform top-hat distribution in pupil space transforms mathematically (by the optical system) to an Airy disk in image space. Laser beams emitted from rotationally symmetric resonators, such as HeNe or YAG lasers with a TEM00 output, have an intensity distribution across the beam which is in the form of a gaussian intensity profile, as shown in Fig. 11.1. A gaussian intensity distribution in pupil space will mathematically transform to a gaussian in image space if the beam is not truncated by the aperture of the optical system, which is, of course, different from the uniform pupil transformation. Note that all of the material in this chapter assumes an aberration-free optical system. It is important to include the effects of lens aberrations in the final assessment of image quality and spot size.

The optical design of systems through which laser beams propagate is, therefore, very different from the design of conventional nonlaser systems used in either the visible or some other wavelength region. First, color correction of the laser-based optical system is much easier

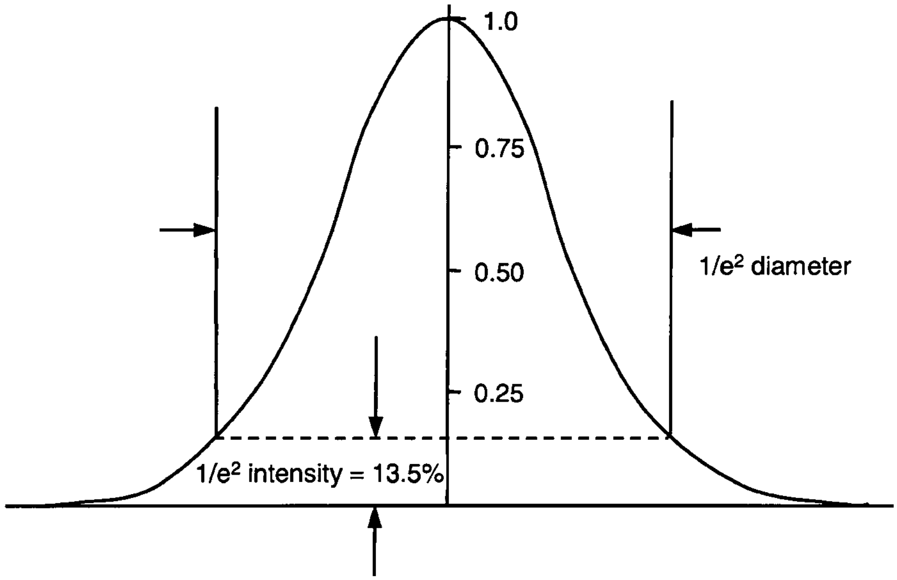


Figure 11.1 Gaussian Intensity Distribution

because the wavelength band of the laser light is extremely narrow. For HeNe lasers fully monochromatic light at 0.63282 μm can be used. Some lasers emit multiple spectral lines, or wavelengths can be changed. In these cases the optics must be designed to cover the functional spectral bandwidth. Laser diodes sometimes have a wavelength shift with temperature, which also must be taken into account by the optics.

Laser systems are often corrected for a small field of view because the laser beam enters the system parallel to the optical axis. Even if the real usable field of view is zero, the optical design must be optimized over a small, yet finite field of view in order to accommodate assembly and alignment tolerances. If one were to design such a system at identically zero field of view, it is possible that the performance may seriously degrade within 1 or 2 mrads off axis. However, there are systems, such as laser scanning systems, where some components of the system have to be designed for a large field of view. There are also systems where it is required to focus a laser beam to a very small spot, which requires the design of diffraction-limited optics with very large numerical apertures. In some cases, the laser beams have high-power densities, and they can cause damage to the optical components. Transparency of the material from which the components are made can be degraded, and the material used for cementing of components and coating of the components can be damaged. The choice of optical materials is thus very important. In some cases, dust particles on the components in the system can also absorb enough energy to damage the surface of the component. Scattering from surface defects is a greater problem in laser systems than in visible and other incoherent systems.

The coherence length of a gaussian laser beam is large, and it appears as if the wavefront emerges from one point. If a gaussian beam is not truncated by the optical system, it emerges from the system as a gaussian beam. The narrow spectral line width of a laser beam and a well-defined wavefront permit very precise focusing and control of the beam.

Beam Waist and Beam Divergence

The beam emitted from a laser in TEM00, or fundamental mode, has a perfect plane wavefront at its beam waist position and a gaussian transverse irradiance profile that varies radially from the axis, which can be described by

$$I(r) = I_0 \exp\left(-2\frac{r^2}{r_0^2}\right)$$

or by the beam diameter

$$I(d) = I_0 \exp\left(-2\frac{d^2}{d_0^2}\right)$$

where I_0 is the axial irradiance of the beam, r and d are radius and diameter of a particular point in the beam, and r_0 and d_0 are the radius and diameter of the beam where irradiance is $(1/e^2)I_0$, or 13.5% of its maximum intensity value on axis. To define the propagation characteristics of a laser beam, the value, r_0, is accepted as a definition of the radial extent of the beam. Finite apertures in the optical system or inside the laser itself, along with diffraction, cause the beam to diverge or converge. There is no such thing as a perfectly collimated beam without any spreading. Spreading of a laser beam is defined by diffraction theory. A laser beam converges to a point where the beam is smallest, called the *beam waist*, and it then diverges from this point with a full angular beam divergence, θ, which is the same as the convergence angle. The beam divergence angle, θ, is the angle subtended by the $1/e^2$ diameter points in the far field, where the irradiance surface asymptotically approaches the full divergence angle, θ, shown in Fig. 11.2. At the location of the beam waist, the laser beam wavefront is flat, and it quickly acquires curvature on both sides of the waist. The beam waist diameter, d_0, depends on the full divergence angle, θ, as

$$d_0 = \frac{4\lambda}{\pi\theta}$$

where λ is the wavelength of the laser beam and θ is in radians. It is important to know that the product of the divergence and the beam waist diameter is constant. The beam diameter grows with the distance, z, from the beam waist according to

$$d(z) = d_0\sqrt{1 + \left(\frac{4\lambda z}{\pi d_0^2}\right)^2}$$

The radius of the wavefront at the beam waist location is infinite. As we move away from the beam waist, it then passes through a minimum at some finite distance, after which it rises again, approaching the value of

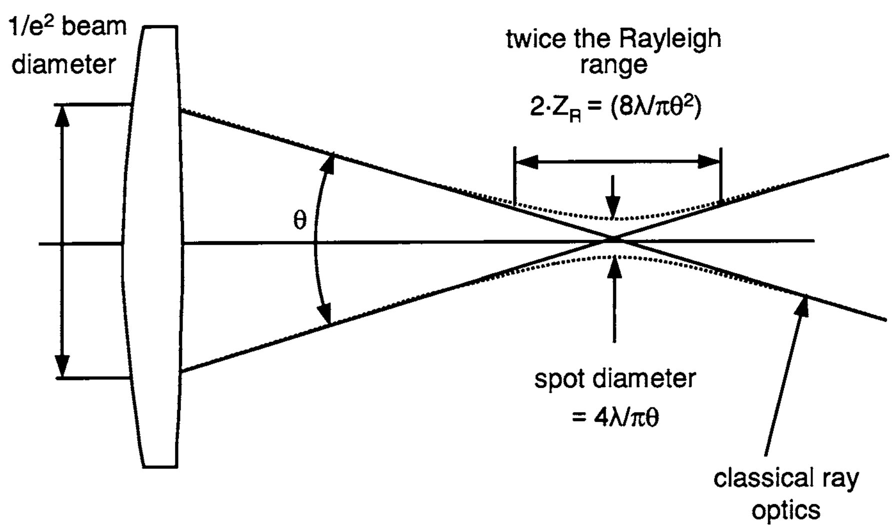


Figure 11.2
Divergence of a Laser Beam in the Far Field

z as z approaches infinity. The radius of the laser beam wavefront can be expressed as

$$R(z) = z\left[1 + \left(\frac{\pi d_0^2}{4\lambda z}\right)^2\right]$$

The minimum value of the wavefront radius occurs at what is known as the *Rayleigh range*, where the beam diameter has the value $\sqrt{2}d_0$. At the extremes of the Rayleigh range, the image diameter is thus 41% larger than at the center of the beam waist. The Rayleigh range is sometimes used as a depth of focus number; however, do keep in mind that this reasonably large increase in spot diameter may not be acceptable. The Rayleigh range as measured from the beam waist is

$$z_R = \frac{d_0}{\theta} = \frac{4\lambda}{\pi\theta^2} = \frac{\pi d_0^2}{}$$

The wavelength of the laser radiation, the beam waist diameter, the beam divergence angle, and the Rayleigh range are four parameters that completely describe a gaussian beam.

Collimation of Laser Beams

There is no perfectly collimated beam, since the beam divergence and the beam waist are defined by diffraction theory. It can be shown that the minimum beam spread between two points at a distance, z, happens when the starting beam waist is equal to

$$d_0 = 2\left(\frac{\lambda z}{\pi}\right)^{1/2}$$

This relationship is equivalent to the one that gives the Rayleigh range, which tells us that the Rayleigh range is the distance inside which a gaussian beam has the minimum spreading. The diameter of the beam at the Rayleigh range is

$$d = \sqrt{2}d_0$$

If we use a beam expander at the output of a laser, we can increase the distance of the minimum spreading. If we additionally adjust the beam expander so that the beam waist is located in the middle of the starting Rayleigh range, then the beam will spread to $\sqrt{2}d_0$ over a distance twice as long.

Propagation of Gaussian Beams and Focusing into a Small Spot

In 1983 S.A. Self derived a simple algorithm for tracing a gaussian beam through an optical system. The formula that he used had a very similar form to the paraxial lens formula that gives the relationship between the object position, focal length of a lens, and the image position. For a gaussian beam, Self calculates the Rayleigh range and the beam waist transformation by each lens in the system:

$$\frac{1}{s + [1/(s - f)]z_R^2} + \frac{1}{s'} = \frac{1}{f}$$

where s is the distance from the lens to the waist on the object side and s' is the distance from the lens to the new waist position. When the incident beam has its waist located in the front focal plane of a positive lens, the emerging beam has its waist at the rear focal plane.

Optical design programs generally use the gaussian beam propagation algorithm derived by A.E. Siegman. These programs usually calculate the radial beam size (semidiameter), the narrowest radial waist, surface coordinates relative to the beam waist, the semidivergence angle, and the Rayleigh range. This assumes the TEM00 fundamental mode, that is, a gaussian irradiance distribution. Lasers may be able to produce a number of other stable irradiance distributions, or modes, but they are not as compact as a gaussian beam, and they all have regions or holes in the beam, that is, where the irradiance drops to zero within the irradiance distribution. The mixed mode beams, defined by the beam quality factor, M, can also be traced in the current generation of optical design programs, in which case the M factor scales the embedded TEM00 gaussian mode.

In many applications, the goal is to focus the laser beam down to a very small spot. If the optical system focusing the beam is diffraction limited, the spot diameter at $1/e^2$ of the peak irradiance is defined by

$$d = \frac{4\lambda f}{\pi d_0}$$

The depth of focus is proportional to the square of the $f/\#$ of the focusing lens. If we define the allowable increase in the diameter of the focused spot, the depth of focus can be calculated using the formula for the spot diameter as the function of the distance from the waist location

$$d(z) = d_0 \sqrt{1 + \left(\frac{4\lambda z}{\pi d_0^2} \right)^2}$$

Truncation of a Gaussian Beam

Let us assume that we have a diffraction-limited lens with a given aperture diameter. The intensity profile of the focused spot is dependent on the intensity distribution of the radiation filling the aperture of the lens. For a uniformly illuminated aperture, the diffraction pattern is the classical Airy disk. It has a central bright spot and progressively weaker rings, with the first dark ring (intensity falls to zero) at a diameter, d:

$$d = 2.44\ \lambda \cdot f/\#$$

When the illumination in the aperture is not uniform, the intensity profile in the focused spot does not have zero-intensity points, and the measure of the spot diameter is usually accepted as the diameter at which the intensity drops to $1/e^2$ of the peak intensity. When a gaussian beam falls onto the aperture of a lens, it may be truncated by the lens aperture. Let us define the truncation ratio as

$$T = \frac{d_0}{D}$$

where d_0 is a gaussian beam diameter measured at $1/e^2$ of the peak intensity and D is the aperture diameter of the lens. In the case of the lens aperture being two times the gaussian beam waist diameter ($T = 0.5$), the beam is truncated only below the intensity level of 0.03%. We can say that the effect of truncation is negligible, and the gaussian beam after transformation by the lens remains gaussian. On the other hand, when T, the truncation ratio, becomes a large number, and only the narrow central portion of the gaussian beam is transmitted through the lens, this case corresponds to the uniform illumination of the lens aperture, and the transmitted beam has the intensity distribution similar to the Airy disk. The beam waist diameter, d_0, given in units of $\lambda \cdot f/\#$ for a few values of

TABLE 11.1

Beam Waist Diameter in Units of $\lambda \cdot f/\#$ for Several Values of the Truncation Ratio

Truncation Ratio T	d_0 at $1/e^2$ Intensity (in units of $f/\#$)	d_0 (Intensity Goes to Zero)	Truncation Intensity Level (%)
$\approx\infty$	1.64	2.44	100
2	1.69	—	60
1	1.83	—	13.5
0.5	2.51	—	0.03

truncation ratio is given in Table 11.1. Two cases of the intensity distribution for $T = \infty$ and $T = 1$ are shown in Fig. 11.3.

In Fig. 11.4, we show how the truncation ratio affects the performance of an otherwise perfect system with a gaussian intensity profile incident onto the entrance pupil of an imaging optical system. The abscissa is the ratio of the physical aperture to the $1/e^2$ beam diameter. The ordinate on the right is the normalized spot radius (normalized to $1/e^2$ spot radius).

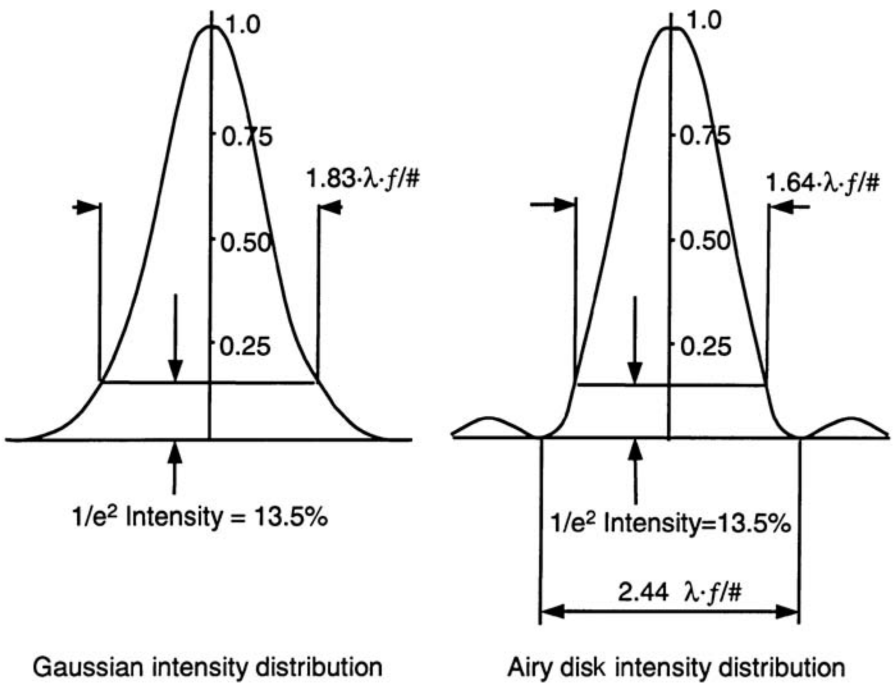


Figure 11.3 Truncation of a Gaussian Beam and the Intensity Distribution at the Image Plane

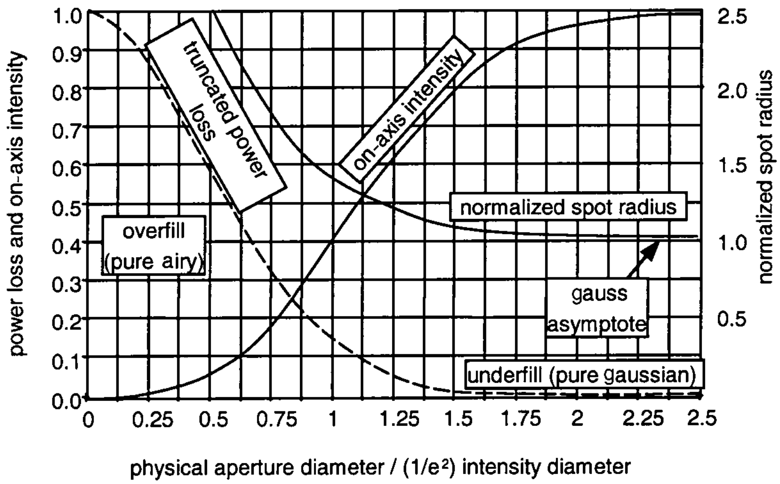


Figure 11.4 Truncated Gaussian Beam Imagery. Aperture Changes Relative to a Constant $1/e^2$ Diameter

The easiest way to understand this somewhat complex data is to think of a constant $1/e^2$ beam diameter of, say, 25 mm. If the physical aperture diameter were 1 m, there would be virtually zero truncation of the beam, and a gaussian intensity distribution in the entrance pupil would transform into a perfect gaussian image profile at the image. The normalized spot radius asymptotically approaches unity as the physical aperture diameter continues to increase. The far right data point in the abscissa in Fig. 11.4 is for the physical aperture being 2.5 times the $1/e^2$ beam diameter. If, on the other hand, the physical aperture diameter were in the order of 3 mm for our 25-mm $1/e^2$ beam diameter, the ratio of the physical aperture diameter to the $1/e^2$ beam diameter would be about 0.125 and the intensity profile would be nearly a top-hat, which means that we will acquire diffraction and the imagery will be close to a classical airy disk pattern. It should be clear that if we truncate the beam at the $1/e^2$ beam diameter, the $1/e^2$ diameter of the image will be approximately 40% larger than an untruncated beam.

The ordinate on the left in Fig. 11.4 is for both the power loss and the on-axis intensity, with the abscissa having the same meaning as before. As the ratio of the physical aperture to the $1/e^2$ beam diameter increases, the on-axis intensity increases, and the loss in power due to truncation decreases.

In Fig. 11.5 we show a similar plot, only here we have the $1/e^2$ beam diameter changing relative to a constant physical aperture diameter. The interpretation and rationale is the same as for the prior data.

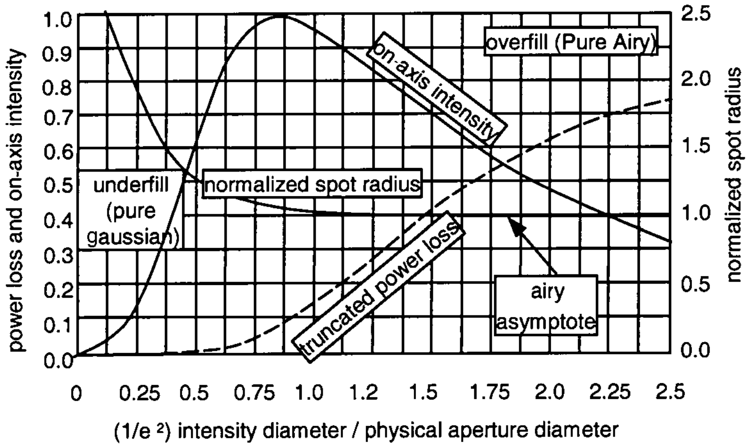


Figure 11.5 Truncated Gaussian Beam Imagery. $1/e^2$ Diameter Changes Relative to a Constant Aperture Diameter

There are several important messages from Figs. 11.4 and 11.5. First, a small beam truncation will affect the $1/e^2$ beam diameter of the resulting image. Truncating the beam to the $1/e^2$ beam diameter will increase the $1/e^2$ spot diameter by approximately 40%. In order to increase the spot diameter by less than 10%, we require a physical aperture about 50% larger than the $1/e^2$ beam diameter. The other data which are of significance are the intensity and power loss associated with a given truncation factor. If our physical aperture were 50% larger than the $1/e^2$ beam diameter, our on-axis intensity would be 80% of that from an untruncated beam.

Application of Gaussian Beam Optics in Laser Systems

Gas lasers have their place in many applications in the large consumer market. HeNe lasers, which emit red light at $\lambda = 632.82$ nm, are used in bar code readers, the printing industry, machine vision, etc. They are used wherever the packaging constraints are not too tight and the use of visible light is convenient. The beam out of the HeNe laser is TEM00 rotationally symmetric gaussian beam, which can be easily transformed with the properly designed optics to the required spot size.

Solid-state lasers can generate very powerful beams. They can emit TEM00 gaussian beams. There are a significant number of types of solid-state lasers that emit in the near IR spectral region. A YAG laser, which emits light at the wavelength of 1.064 μm, can be frequency doubled to produce green light at 532 nm. It can also be frequency tripled or quadrupled. YAG lasers are used in industrial applications where high power is needed, such as writing on metal, welding, cutting, hole drilling, etc. One disadvantage of solid-state lasers is that they are generally expensive. One field that makes an extensive use of lasers is medicine, especially in the area of diagnostics, cancer treatment, and eye surgery. The laser wavelengths used range from the UV below 200 nm to 10.6 μm in the far infrared. CO_2 gas lasers emit radiation at 10.6-μm wavelength, and they are widely used both in medical and industrial applications.

UV lasers are used in lithography to achieve the submicrometer imagery needed to make integrated circuit chips. The minimum spot size is determined by the wavelength and the numerical aperture of the focusing optics. The shorter the wavelength, the smaller the spot to which the laser beam can be focused. This pushes the microprocessor industry to use increasing shorter wavelengths. Currently, a common wavelength is 193 nm.

Laser diodes are undergoing an extremely rapid development. Advantages of laser diodes include their low cost and small physical size. They are also available in a wide range of wavelengths. Laser diode devices provide continuous output power or may be analog or digitally modulated. Pulsed laser diodes typically operate with pulses shorter than 100 ns. Laser diodes can be packaged in the form of a linear diode array or a two-dimensional array. Laser diodes may be temperature tuned by approximately 0.3 nm/°C. Applications of laser diodes range from telecommunications, data communications, sensing, thermal printing, laser-based therapeutic medical systems, and, satellite telecommunications to diode pumping of solid-state lasers.

The primary optics-related disadvantage of laser diodes is that they emit nonrotationally symmetric beams, which are more difficult to collimate or focus into a small spot. Most beams from laser diodes have a gaussian intensity profile along one axis, often called the *fast axis,* and a nongaussian intensity profile along the *slow axis* perpendicular to it. The beam diverges much faster in the fast axis than in the slow axis. Achieving collimation of the beams from laser diodes is not a trivial task because of the lack of the rotational symmetry in the beam. It requires anamorphic optics, in most cases treating the fast and the slow axes separately. A good way to collimate a beam from a laser diode is to use two crossed glass or gradient index fibers, perpendicular to the optical axis of the laser beam shown in Fig. 11.6, each fiber having a different focal length. This is an elegant solution because of the small packaging. Two perpendicular fibers have small focal lengths. The fiber that collimates the fast axis has a focal length of a few hundred micrometers, and can be a gradient index fiber with a radial gradient index distribution, or a planohyperbolic homogeneous fiber. The fiber that collimates the slow axis has a longer focal length. In this way, the output of most laser diodes can be transformed into near rotationally symmetric beams. The beam can then be focused with a rotationally symmetric lens into a single-mode or a multimode fiber core that transmits the laser signal along the fiber axis.

A very convenient way of transmitting laser beams is through fibers. The laser beam from a laser diode can be coupled into a fiber with a coupling efficiency as high as 90% and transformed to a clean beam, easier to collimate at the output of the fiber. In telecommunications, the laser light-carrying signals are transmitted through hundreds of kilometers of fiber.

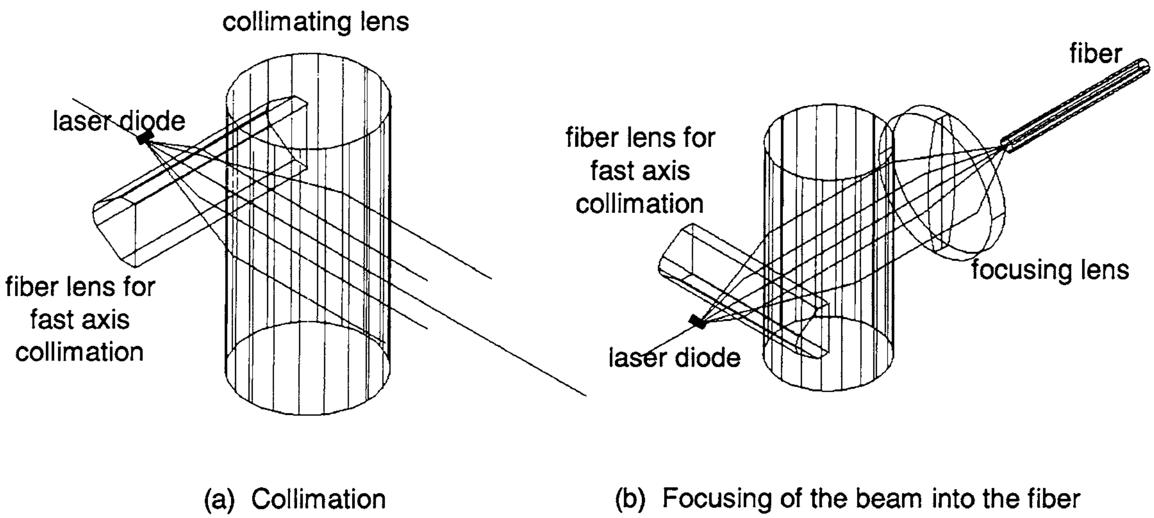


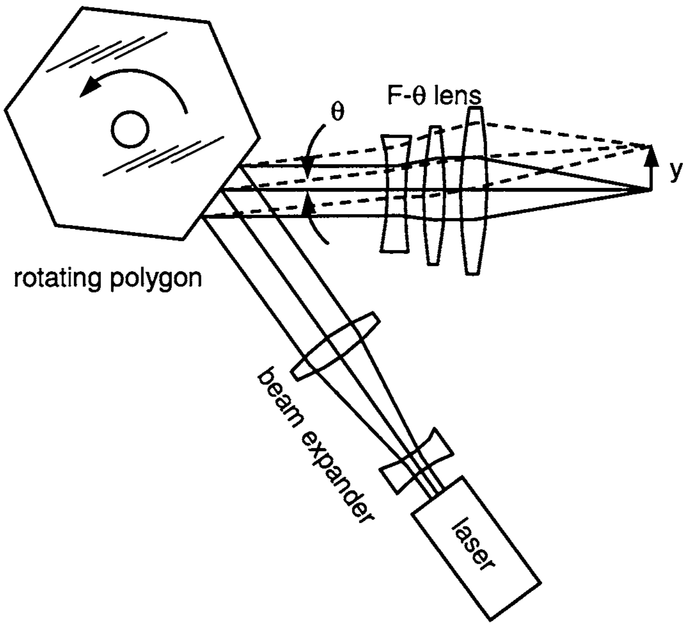
Figure 11.6
Collimation and Focusing of the Beam from the Laser Diode

F-θ Lenses in Laser Scanners

Laser scanners represent one of the more common applications of gaussian beam optics, and a very special lens form is required in these laser scanning systems. Figure 11.7 shows a very generic laser scanning system. An HeNe or similar laser beam is first expanded by a beam expander, and the expanded laser beam is directed toward a multifaceted polygon scan mirror. As the mirror rotates at a constant rotational speed, angle θ changes linearly.

Most lenses follow the relationship that the image height $Y = f \tan\theta$. This is literally true for lenses with zero distortion. If such a lens were used in our laser scanner, then the velocity of the scanned laser spot would increase in proportion to the tangent of the scan angle. Since most laser scanners require a linear spot velocity, conventional lenses are not viable. Lenses which follow the relationship, $Y = f\,\theta$, will produce a linear scan velocity, and these lenses are called *F*-theta, or *F*-θ, lenses. In effect, they are lenses in which negative or barrel distortion is intentionally introduced in order to counteract the increased image

Figure 11.7
Laser Scanner with *F*-θ lens



velocity in conventional low-distortion optics. Fortunately, these forms of remote-aperture stop lenses (the stop is on the polygon facet) tend to inherently have negative distortion. It is critical, however, to assure uniform spot size through scan, and this is sometimes challenging. In order to loosen the tolerance on pyramidal error of the polygon, the collimated beam from the laser is often focused with a cylindrical lens to a line on the polygon facet. This requires an anamorphic F-θ lens to focus the beam to a spot. This form of optical system is used in laser printers.